Noise, information and elements of matter

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1 Introduction

The purpose of this paper is to discuss recent developments in the mechanisms of communicating information. We briefly review the relevant history and discuss new noise and information mechanisms and how these are represented and communicated.

1.1 Terminology

The terminology with respect to matter, energy, particles and waves is not a settled matter. Einstein's [4] proposal of the photon which introduced quantized elements of nature is widely accepted, but there seems not to be agreement on what to call the basic elements. The terms "particles" or "waves" are commonly used. But this often also reflects a distinction between those who argue that the fundamental unit of nature is the particle versus those claiming that wave-like behavior is basic.

Setting aside the debate about what is fundamental, candidate alternative terms include: quanta, waves, particles, or grains,

In French, deBroglie uses "grain" for the elementary object in his theory, and develops the connection of "grains" and waves. In the English translation of deBroglie's *Matter and Light*[3] grain is translated as "corpuscle".

However, here we will use use the, we hope, neutral term "element of matter."

2 Information

Classical information theory is based on Shannon Information which provides the theoretical limit for the bit rate in a channel of fixed finite bandwidth. However, it does not address the question of how information may be encoded in physical media. Nor does it address the question of how to best obtain information about a physical system.

In contexts other than Shannon Information Theory, information is is usually defined as the acquisition of some "fact." The Oxford dictionary definition of information is consistent with this interpretation. This definition suggests a change of state from knowing some set of facts to knowing or forgetting some facts. Formally, this can be viewed as a change in entropy. However, it is not necessary that additional information reduces entropy.

2.1 Signals

The original papers on the representation of signals are Ville [15] and Gabor [5]. Gabor [6] also discusses the nature of information processing with coherent light. This represents an important step leading, of course, to holography and laser optics. However, Gabor assumed that photons have 2 degrees of freedom.

This is consistent with the results of Huygens but not of Poynting. Huygens[7] observed that the brightness of light from stars was not a function of the distance to the star. Thus, light, in free space, must be coherent. Poynting[9] pointed out that a photon possesses angular orbital momentum (OAM) and this leads to an indefinite number of degrees of freedom per photon. Recent experiments confirm this result for an increasingly large number of degrees of freedom, thus increasing the available information rate for optical transmission by orders of magnitude.

A good example of the use of OAM is Nagali [8].

In addition, Gabor [6] pointed out that a finite vector space can contain an indefinitely large number of approximately orthogonal vectors. This result has been used by Wild [19] to produce more precise estimates from antenna arrays.

3 Noise

It has been widely assumed that noise is, in some sense, a property of the physical system which is under examination. The "Heisenberg Uncertainty Principle" is presented as a property of this physical system. The "uncertainty" is therefore inherent in the physical system. However, this suggests that if it were possible to conduct a measurement for an infinitely long period of time, the result would still be "uncertain" by some fixed amount. The behavior of this "error" as a function of the length of the time measurement is at best unclear.

This view determines limits on what we can discover about the physical world independent of the mechanisms which are available for measurement.

This "uncertainty principle" is a well-known mathematical property of non-commuting operators. In information theory it is referred to as Young's Inequality. In statistics it is Fisher Information and the Cramer-Rao lower bound. (See Wikipedia Cramer-Rao bound.) This property bounds the

amount of information that can be obtained about an observable system. It is fundamental to information theory, statistics, and other disciplines which concern systems which are characterized by unbounded extent but finite variability.

3.1 Error

> "The present treatment of error is unsatisfactory and ad hoc. It is the author's conviction, voiced over many years, that error should be treated by thermodynamical methods, and be the subject of a thermodynamical theory, as information has been, by the work of L. Szilard [14] and C. E. Shannon [13]. The present treatment falls far short of achieving this, but it assembles, it is hoped, some of the building materials, which will have to enter into the final structure."

J. von Neumann(1952) [16] Section 5.2

In addition, in a letter dated February 14, 1945, to F. B. Silsbee, of The Philosophical Society of Washington[1], concerning his planned "Joseph Henry Lecture" to be given on March 17 von Neumann provided this title and abstract:

> ... the title "Causality, Statistics and Quantum Mechanics" suits me very well. ... I would phrase an abstract as follows:
>
> Quantum Mechanics is more closely connected with Probability and Statistics than any other branch of Physics. An analysis shows that the relation is of a deeper nature than even in classical statistical mechanics. The structure of logics and the relation of strict logics to probability is also essentially affected by the recognition of the new features of Quantum Mechanics. Finally, Quantum Mechanics provides an example for the possibility of building a complete and self consistent system, in which causality does not hold in the conventional sense.

The talk was apparently given but after further correspondence on July 2, 1945, von Neumann wrote:

> It is with great regret that I am writing these lines to you, but I simply cannot help myself. In spite of very serious attempts to

[1] See Redei [11] pg. 217

> write the article on the "Logics of Quantum Mechanics" I find it completely impossible to do it at this time.

He then states that he did complete a paper on the subject with Birkhoff in 1936 [2]. But he also explains the exceptionally great difficulty he had had while attempting drafts, all of which appeared to him to be unsuitable. Elsewhere he mentioned that he hoped to address this topic when he took up his offered Professorship at UCLA. His death in 1957 prevented this.

3.2 Fisher information

Fisher information measures the amount of information that an observed variable X provides about a hidden variable θ. This is a formal result without reference to any physical properties or processes. It is one of the fundamental results in information theory. This, and the related results, depend on the probability distribution of the observed variable, usually assumed to be Gaussian, being known.

4 Information Limit and Coherence

In cases where this information limit applies there is no reason to try to determine the cause a priori. This is a purely formal result concerning measurements of variables which have unbounded range, but bounded second moment (in physical systems, that is energy). In the case of the physical systems that we describe here it seems appropriate to point out that the information limit is due to the measurement mechanism and not, as is usually assumed, to the physical system itself. Specifically, the coherence of light in free space continues indefinitely. See Huygens [7]. Only if it is disturbed by some energy-matter interaction, such as a measurement, is there any decoherence. There is nothing "uncertain" about the light itself. The sampling mechanism which detects the light determines how much noise is introduced. Obviously, the reduction of this noise has been a continuing challenge of modern quantum- and astro-physics.

The understanding of the behavior of time signals was greatly improved by Ville [15] and Gabor [5]. It was then believed [5] that a single photon provides a single bit of information. However, this was shown to be over simplified as long ago as 1909 by Poynting [10] in theory and experimentally by Beth in 1936 [1]. Beth measured the torque:

$$L = -(A/4\pi)^2 n\lambda \sin 2\theta (\sin 2\Delta_2 - \sin 2\Delta_1).$$

(See eq. 3 of Beth [1]) which had been previously been considered to be be too small for experimental detection.

However, using modern techniques, the number of bits that can be encoded and detected in the orbital angular momentum modes of a photon is quite substantial and increasing as techniques improve.

This is currently being exploited in advanced optical communications systems, thus increasing the data transmission rates by many factors. A recent example of this is Nagali [8]. The practical number of bits appears to be an "engineering" issue concerning reliability and complexity of the encoding and decoding mechanisms. It apparently is not known whether biological systems sense more than one bit per photon, or if so how this information would be interpreted.

4.1 Noise Source

In addition, it has been assumed that the "noise" in the communication system is introduced in the channel, not in the transmitter or receiver. Thus, Resnikoff [12] (pg. 399) states:

A communication system, according to Claude Shannon, consists of an information source, a transmitter, a noise source, a receiver, and a destination. The task of the transmitter is to encode the message provided by the information source and to modulate it onto the communication channel. The receiver detects, demodulates and decodes the message for the destination. The noise source acts on the signal while it passes from the transmitter to the receiver, that is, while the signal is in the communication channel. In fact, the transmitter and receiver will also be sources of noise but we will ignore this aspect of the system.

At the time, this was an appropriate description. However, using today's optical communications the reverse is true. The "channel" is coherent (noiseless (See [17])). But, most prominently, the receiver introduces noise as an essential property of the receiver measurement.

5 Communication

For coherent light to cause communication to an "observer" some event must take place that cannot be measured exactly. In addition, there must be some means for the observer to know what the communicated patterns represent.

This has always been the case as in the communication of the characters composed into words from a known artificial or natural language.

6 Quantum Photonics

While light in free space is perfectly coherent, light traveling in some medium interacts with that medium with some loss of coherence. In any case, the practical use of OAM optical systems through, for example, fiber optic channels, requires a means of detecting and correcting any errors. Recent work [18] indicates a means of doing this. Thus, in practice, channels using OAM and error management can operate at close to the channel capacity of communication in free space.

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